CONTACT MODEL: HERTZ-MINDLIN

This is soft-sphere model which is used to calculate particle-particle or particle wall contact. In both cases calculations are similar, however, when particle-wall contact is calculated, then particle radius and mass are considered as equivalent radius R^* and mass M^*.

Velocities:

$$\bar{v}_{rel} = \bar{v}_2 - \bar{v}_1 + \bar{\omega}_1 \times \bar{r_c} - \bar{\omega}_2 \times \bar{r_c}_2$$

$$\bar{v}_{rel,n} = \bar{r}_n \cdot (\bar{r}_n \cdot \bar{v}_{rel})$$

$$\bar{v}_{rel,t} = \bar{v}_{rel} - \bar{v}_{rel,n}$$

Additional parameters:

$$\alpha = \frac{\ln(e)}{\sqrt{\pi^2 + \ln^2(e)}}$$

$$R^* = \frac{r_1 \cdot r_2}{r_1 + r_2}$$

$$M^* = \frac{m_1 \cdot m_2}{m_1 + m_2}$$

$$E^* = \left(\frac{1 - v_1^2}{E_1} + \frac{1 - v_2^2}{E_2}\right)^{-1}$$

Normal force:

$$\xi_n = r_1 + r_2 - |O_2 - O_1|$$

$$k_n = 2E^* \sqrt{\xi_n \cdot R^*}$$

$$\bar{F}_n = -\bar{r}_n \cdot \frac{2}{3} \xi_n \cdot k_n - \bar{r}_n \cdot sgn\left(\bar{v}_{rel,n} \cdot \bar{r}_n\right) \cdot 1.8257 \cdot \alpha \cdot |\bar{v}_{rel,n}| \cdot \sqrt{k_n \cdot M^*}$$

Tangential force:

$$\Delta\bar{\xi}_t = \bar{v}_{rel,t} \cdot \Delta t$$

$$k_t = 8 \cdot G^* \cdot \sqrt{R^* \cdot \xi_n}$$

$$\Delta\bar{F}_t = \left[k_t \cdot \Delta\bar{\xi}_t - 1.8257 \cdot \alpha \cdot \bar{v}_{rel.t} \cdot \sqrt{k_t \cdot M^*}\right]$$

$$\bar{F}_{t,pr}^{cor} = \bar{F}_{t,pr} - \bar{r}_n \cdot \left(\bar{r}_n \cdot \bar{F}_{t,pr}\right)$$

$$\overline{F}_{t,pr}^{cor} = \overline{F}_{t,pr}^{cor} \cdot |\overline{F}_{t,pr}|/|\overline{F}_{t,pr}^{cor}|$$

$$\overline{F}_t = \overline{F}_{t,pr}^{cor} + \Delta\overline{F}_t$$

if

$$|\overline{F}_t| > \mu_{sl} \cdot |\overline{F}_n|$$

then

$$\overline{F}_t = \mu_{sl} \cdot |\overline{F}_n| \cdot \frac{\overline{F}_t}{|\overline{F}_t|}$$

Rolling friction:

$$\overline{M}_{ro,1} = -\mu_{ro} \cdot |\overline{F}_n| \cdot r_1 \cdot \frac{\overline{\omega}_1}{|\overline{\omega}_1|}$$

$$\overline{M}_{ro,2} = -\mu_{ro} \cdot |\overline{F}_n| \cdot r_2 \cdot \frac{\overline{\omega}_2}{|\overline{\omega}_2|}$$

Summarized forces and moments acting on particle (wall):

$$\overline{F}_{tot} = \overline{F}_n + \overline{F}_t$$

$$\overline{F}_1 = \overline{F}_n + \overline{F}_t$$

$$\overline{F}_2 = -\overline{F}_n - \overline{F}_t$$

$$\overline{M}_{tot,1} = \overline{r}_n \times \overline{F}_t \cdot r_1 + \overline{M}_{ro,1}$$

$$\overline{M}_{tot,2} = -\overline{r}_n \times \overline{F}_t \cdot r_2 + \overline{M}_{ro,2}$$

Literature

Hertz H. (**1882**). Über die Berührung fester elastischer Körper. *Journal die reine und angewandte Mathematik, 92, 156-171.*

Tsuji Y., Tanaka T., Ishida T. (**1992**). Lagrangian numerical simulation of plug flow of cohesionless particles in horizontal pipe. *Powder Technology, 71 239-250.*

Symbol	Description
$\Delta\bar{\xi}_t$	Tangential displacement in the current step [m]
e	Restitution coefficient [-]
E^*	Equivalent Young's modulus [Pa]
E_1, E_2	Young's moduli of contact partners [Pa]
$\bar{F}_n, \bar{F}_t$	Force in normal and tangential directions [N]
$\bar{F}_{t,pr}$	Tangential force on previous iteration [N]
G^*	Equivalent shear modulus [Pa]
$\bar{M}_{ro}$	Moment due to the rolling friction [N]
m_1, m_2	Particle masses [kg]
M^*	Equivalent mass [kg]
O_1, O_2	Centers of contact partners [m]
μ_{ro}, μ_{sl}	Coefficient of rolling friction and sliding friction [-]
$\bar{v}_{rel}$	Relative velocity [m/s]
$\bar{v}_1, \bar{v}_2$	Translational velocities of contact partners [m/s]
r_1, r_2	Particle radii [m]
R^*	Equivalent radius [m]
$\bar{r}_c$	Contact vector [m]
$\bar{r}_n$	Normalized contact vector [-]
$\bar{\omega}_1, \bar{\omega}_2$	Rotation velocities of particles [rad/s]
ξ_n	Normal overlap [m]