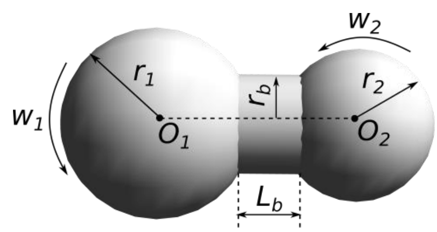
SOLID BOND MODEL: ELASTIC BOND



Bond properties:

$$\boldsymbol{L_b(t) = |O_2 - O_1|}$$

$$L_{init} = L_b(t = 0)$$

$$I = \frac{\pi \cdot R_b^4}{4}$$

$$J = \frac{\pi \cdot R_b^4}{2}$$

$$A_b = \pi \cdot R_b^2$$

Contact vector:

$$\bar{r_c} = O_2 - O_1$$

$$\bar{r_n} = \frac{O_2 - O_1}{|O_2 - O_1|}$$

Velocities:

$$\bar{v}_{rel} = \bar{v}_2 - \bar{v}_1 - \frac{(\bar{\omega}_1 + \bar{\omega}_2) \times \bar{r_c}}{2}$$

$$\bar{\omega}_{rel} = \bar{\omega}_1 - \bar{\omega}_2$$

$$\bar{\omega}_{rel,n} = \bar{r_n} \cdot (\bar{r_n} \cdot \bar{\omega}_{rel})$$

$$\bar{v}_{rel,n} = \bar{r_n} \cdot (\bar{r_n} \cdot \bar{v}_{rel})$$

$$\bar{\omega}_{rel,t} = \bar{\omega}_{rel} - \bar{\omega}_{rel,n}$$

$$\Delta\bar{\delta}_{\omega n,b} = \bar{\omega}_{rel,n} \cdot \Delta t$$

$$\Delta\bar{\delta}_{\omega t,b} = \bar{\omega}_{rel,t} \cdot \Delta t$$

$$\Delta\bar{\delta}_t = \vec{v}_{rel,t} \cdot \Delta t$$

Force and moment in normal direction:

$$\bar{F}_{n,b} = \bar{r}_n \times (L_b - L_{init}) \cdot \frac{E}{L_{init}} \cdot A_b$$

$$\bar{M}_{n,b} = T \cdot \bar{M}_{n,b} + \Delta\vec{\delta}_{\omega n,b} \cdot \frac{E}{2L_{init}(1+\nu)} \cdot J$$

In some literature sources $\bar{M}_{n,b}$ is also named as torsional moment (Shen et al., 2016)

Force and moment in shear direction:

$$\bar{F}_{t,b} = T \cdot \bar{F}_{t,b} + \Delta\bar{\delta}_t \cdot \frac{E}{2L_{init}(1+\nu)} \cdot A_b;$$

$$\bar{M}_{t,b} = T \cdot \bar{M}_{t,b} + \Delta\vec{\delta}_{\omega t,b} \cdot \frac{E}{L_{init}} \cdot I$$

In some literature sources $\bar{M}_{t,b}$ is also named as rolling or bending moment (Shen et al., 2016)

Total moment:

$$\boldsymbol{M_{tot}} = \boldsymbol{M_{t,b}} + \boldsymbol{M_{n,b}} + \frac{\boldsymbol{r_c}}{\boldsymbol{2}} \cdot \boldsymbol{F_{t,b}}$$

Breakage criteria:

$$\frac{F_{n,b}}{A_b} + \frac{M_{t,b} \cdot R_b}{I} = \sigma_{max}$$

$$\frac{F_{t,b}}{A_b} + \frac{M_{n,b} \cdot R_b}{J} = \tau_{max}$$

Some of materials do not break during compression. To consider this the option *Compressive Breakage* can be used. If this option is disabled, then breakage condition for normal strength (σ_{max}) is analyzed only when the length of the bond L_b is larger than initial length L_{init} (bond situated in elongated state).

Literature

Dosta M., Antonyuk S., Heinrich S. (**2013**). Multiscale simulation of agglomerates breakage in fluidized beds*, Industrial Engineering Chemistry Research 52, 11275-11281.*

Kozhar S., Dosta M., Antonyuk S., Heinrich S., Gilson L., Bröckel U. (**2015**). DEM simulations of amorphous irregular shaped micrometer-sized titania agglomerates at compression*, Advanced Powder Technology 26, 767-777.*

Potyondy D.O., Cundall P.A. (**2004**). A bonded-particle model for rock, Int. J. of Rock Mechanics and Mining Sci. 41, 1329-1364.

Shen Z., Jiang M., Thornton C. (**2016**). DEM simulation of bonded granular material. Part I: Contact model and application to cemented sand, Computers and Geotechnics 75, 192-209.

Symbol	Description
$\boldsymbol{A_b}$	Cross-cut surface of the bond [m^2]
$\Delta\bar{\boldsymbol{\delta}}_{\boldsymbol{\omega n,b}}, \Delta\bar{\boldsymbol{\delta}}_{\boldsymbol{\omega t,b}}$	Increment of displacement (in current step) between particles in contact point due to the rotational velocities [m]
$\Delta\bar{\boldsymbol{\delta}}_{\boldsymbol{t}}$	Tangential displacement in the current step [m]
$\boldsymbol{E}$	Young modulus for particle or bond [Pa]
$\boldsymbol{I}, \boldsymbol{J}$	Moments of inertia of the bond [m^3]
$\boldsymbol{L_b}$	Current bond length
$\boldsymbol{L_{init}}$	Initial bond length [m]
$\boldsymbol{O_1}, \boldsymbol{O_2}$	Centers of contact partners [m]
$\boldsymbol{r_1}, \boldsymbol{r_2}$	Particle radii [m]
$\boldsymbol{r_b}$	Bond radius [m]
$\boldsymbol{T}$	Transformation matrix (to consider rotation of the bond in the global coordinate system) [-]
$\bar{\boldsymbol{\omega}}_{\boldsymbol{1}}, \bar{\boldsymbol{\omega}}_{\boldsymbol{2}}$	Rotation velocities of particles [rad/s]